Enhancement of ferromagnetism by *p*-wave Cooper pairing in superconducting ferromagnets

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In superconducting ferromagnets for which the Curie temperature T_m exceeds the superconducting transition temperature T_c, it was suggested that ferromagnetic spin fluctuations could lead to superconductivity with *p*-wave spin-triplet Cooper pairing. Using the Stoner model of itinerant ferromagnetism, we study the feedback effect of the *p*-wave superconductivity on the ferromagnetism. Below T_c, the ferromagnetism is enhanced by the *p*-wave superconductivity. At zero temperature, the critical exchange interaction value for itinerant ferromagnetism is reduced by the strength of the *p*-wave pairing potential, and the magnetization increases correspondingly. More important, our results suggest that once the ferromagnetism is established, T_m is unlikely to ever be below T_c. For strong and weak ferromagnetism, three and two peaks in the temperature dependence of the specific heat are, respectively, predicted, the upper peak in the latter case corresponding to a first-order transition.

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Due to the strong interplay between conventional superconducting (SC) and ferromagnetic (FM) states, the exploration of their possible coexistence in the same crystal might have seemed fruitless, but has nevertheless attracted a great deal of interest recently. This possible coexistence was first proposed by Ginzburg more than 50 years ago.[1] Several years later, Larkin and Ovchinnikov[2] and Fulde and Ferrell[3] independently developed a microscopic theory of this coexistence in the presence of a strong magnetic field, based upon a spatially inhomogeneous SC order parameter, presently referred to as the FFLO state. Meanwhile, Berk, and Schrieffer suggested that conventional *s*-wave superconductivity in the paramagnetic phase above the Curie temperature T_m is suppressed by critical ferromagnetic fluctuations near to T_m.[4] However, more recent calculations showed that conventional *s*-wave superconductivity can form in the weakly FM regime close to a quantum phase transition.[5] In addition, Fay and Appel predicted that *p*-wave superconductivity could arise in itinerant ferromagnets.[6] Their pioneering work indicated that longitudinal ferromagnetic spin fluctuations could result in a *p*-wave "equal-spin-pairing" SC state within and just outside the FM phase.

Experimentally, a major development occurred with the observation by Saxena *et al.* that UGe_2, nominally an itinerant FM compound, undergoes an SC transition at low T_c values under high pressure.[7] An SC state has since been found in the three other itinerant ferromagnets UIr, URhGe, and UCoGe.[8–16] A recent brief discussion of the latest developments was given by de Visser.[17] With the possible exceptions of the high-pressure and high-field regimes of UCoGe[18] and the re-entrant regime of URhGe,[14] the regime of the SC phase appears completely within that of the FM phase, suggesting a cooperative effect between the SC and FM states.

These experimental achievements have stimulated renewed theoretical interest in the subject. Recently, a large effort has been devoted to the understanding of the underlying physics of the coexisting SC and FM states, with a focus upon the SC pairing mechanism and the orbital symmetry of the SC order parameter. Although earlier works by Suhl and Abrikosov suggested that an *s*-wave pairing interaction between conduction electrons could be mediated by ferromagnetically ordered localized spins, such as by impurities,[19,20] recent studies of these four SC ferromagnets have assumed that the itinerant electrons involved in both the FM and SC states are within the same band.[21–26] Some of these studies assumed conventional *s*-wave pairing. For example, Karchev *et al.* studied an itinerant electron model in which the same electrons are responsible for both the FM and SC states.[21] In that study, the Cooper pairs were assumed to be in a spin-singlet state, and the ferromagnetism was described within the Stoner model. However, the resulting SC ferromagnetic state was shown to be energetically unfavorable when compared to the conventional, nonmagnetic SC state.[22] A possible exception to this incompatibility could occur if the magnetic instability were to arise from a dynamic spin exchange interaction, as discussed by Cuoco *et al.*[23] On the other hand, a number of other workers avoided the likely incompatibility of the SC and FM states by assuming for simplicity a spin-triplet SC order parameter with *p*-wave orbital symmetry.[24–26] Kirkpatrick *et al.* indicated that a *p*-wave SC state meditated by ferromagnetic spin fluctuations is more likely to coexist within the Heisenberg FM phase regime than within the paramagnetic phase regime.[24] Machida and Ohmi studied the properties of a *p*-wave SC ferromagnet phenomenologically.[25] More recently, a microscopic model of the coexistence of a nonunitary spin-triplet SC state with a weakly itinerant FM state was developed by Nevidomskyy.[26]

The present nature of the SC state coexistent with the FM state in these ferromagnetic superconductors is still somewhat controversial, although increasingly, additional experiments on UGe_2, UIr, and especially upon URhGe and UCoGe have provided increasing support for a spin-triplet state rather than a spin-singlet one.[11–18] We note that the upper critical field in the "regular" low-field regime of URhGe quantitatively fits the temperature dependence of the completely broken symmetry p-wave polar state in all three field directions,[12,27] indicating weak spin-orbit coupling, as for UPt_3.[28,29] In the reentrant regime of URhGe and in the combined "regular" and reentrant field regime of UCoGe, the upper critical field violates the Pauli paramagnetic limit in at least two directions by more than an order of magnitude.[14–16] In addition, ^{73}Ge nuclear quadrupole resonance experiments on UGe_2 indicated that a highly anisotropic gap opened up only on the up-spin band, but no measurable gap appeared on the down-spin band.[13] Both of these results are strongly suggestive of a parallel-spin SC state.

Most theoretical studies have focused primarily on the effect of the established ferromagnetism upon the nature of the coexistent superconductivity, as summarized above. However, to fully understand the interplay between the SC and FM states when they coexist, one should also study the feedback effect of the superconductivity upon the ferromagnetism itself.

Here we study explicitly the effects of the p-wave pairing on the FM ordering, using the Stoner model of itinerant ferromagnetism, which neglects spin-orbit coupling, as the starting point. We calculate the critical exchange constant U_c, the magnetization m, and the two parallel-spin p-wave gap function magnitudes, $\Delta_\pm$, respectively, as functions of the pair-interaction strength V. We also discuss finite-temperature properties, including the temperature T dependencies of these order parameters and the specific heat $C(T)$.

Our results show that p-wave triplet Cooper pairing can enhance the ferromagnetism in superconducting ferromagnets. This enhancement is most prominent for the magnetic exchange interaction U very near to $U_c(0)$, the critical value for the strength of the exchange interaction required for the onset of ferromagnetism in the absence of the p-wave pairing interaction V. With finite V, $U_c(V)$ is reduced and the ferromagnetic order parameter increases in magnitude with increasing V. The T dependencies of the magnetic and parallel-spin superconducting order parameters show that the Curie temperature is unlikely to ever be lower than the upper SC transition temperature, which prediction may be relevant to experiments on all four known ferromagnetic superconductors. Our results support the possible coexistence of p-wave superconductivity with a ferromagnetic state. The specific heat $C(T)$ exhibits two peaks for weak ferromagnetism in the coexistence state, with a first-order transition at the combined ferromagnetic and upper p-wave SC transition, and a lower second-order p-wave SC transition. For strong ferromagnetism, the specific heat exhibits three second-order transitions.

We take the Hamiltonian for the ferromagnetic superconductor to have the form

$$H_{\mathrm{FM+SC}} = \sum_{\mathbf{k},\sigma} (\epsilon_{\mathbf{k}} - \mu - \sigma M) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{1}{2\mathcal{V}} \sum_{\substack{\mathbf{k},\mathbf{k}'\\ \sigma,\sigma'}} V_{\mathrm{SC}}(\mathbf{k},\mathbf{k}') c^\dagger_{\mathbf{k},\sigma} c^\dagger_{-\mathbf{k},\sigma'} c_{-\mathbf{k}'\sigma'} c_{\mathbf{k}'\sigma}, \quad (1)$$

where $\sigma = \pm$ represent the single-particle spin states, and the single-quasiparticle part of H comprises the Stoner model for itinerant electrons, where $\epsilon_{\mathbf{k}}$ is the nonmagnetic part of the quasiparticle dispersion, μ is the chemical potential and $M = U(\langle n_+\rangle - \langle n_-\rangle)/2$ is the magnetic molecular field with U the Stoner exchange interaction, and $\mathcal{V}$ is the sample volume. The pairing potential is taken to have the p-wave form,[30] $V_{\mathrm{SC}}(\mathbf{k},\mathbf{k}') = -V\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}'$. In weak coupling theory, V is nonzero and assumed to be constant only within the narrow energy region $|\epsilon - \epsilon_F| \le \omega_c$ near to the Fermi energy ϵ_F, where ω_c is the energy cutoff.

Because of the pair-breaking effects of the strong exchange field in ferromagnets, we assume that only parallel-spin Cooper pairs can survive. Thus we set the p-wave antiparallel-spin gap function $\Delta_0 = 0$ and retain the two gap functions $\Delta_\pm(\mathbf{k})$ with parallel-spin states $m_S = \pm 1$. The SC order parameters are assumed to have the following p-wave symmetry,[30] $\Delta_{\pm 1}(\mathbf{k}) = (\hat{\mathbf{k}}_x + i\hat{\mathbf{k}}_y)\Delta_\pm$.

The Hamiltonian is treated via the Green's function method within the mean-field theory framework. In addition to the normal Green's function $\mathcal{G}_\sigma(\mathbf{k},\tau-\tau') = -\langle T_\tau c_{\mathbf{k}\sigma}(\tau) c^\dagger_{\mathbf{k}\sigma}(\tau')\rangle$, the anomalous Green's function describing the pairing of electrons should be introduced, $\mathcal{F}_\sigma(\mathbf{k},\tau-\tau') = \langle T_\tau c_{\mathbf{k}\sigma}(\tau) c_{-\mathbf{k}\sigma}(\tau')\rangle$. Using the standard equation of motion approach, the Green's functions are derived to be

$$\mathcal{G}_\pm(\mathbf{k},ip_n) = \frac{-(ip_n + \epsilon_{\mathbf{k}} \mp M)}{p_n^2 + (\epsilon_{\mathbf{k}} \mp M)^2 + |\Delta_{\pm 1}(\mathbf{k})|^2},$$

$$\mathcal{F}_\pm(\mathbf{k},ip_n) = \frac{\Delta_{\pm 1}(\mathbf{k})}{p_n^2 + (\epsilon_{\mathbf{k}} \mp M)^2 + |\Delta_{\pm 1}(\mathbf{k})|^2}, \quad (2)$$

where the p_n are the Matsubara frequencies, and the FM and SC order parameters are respectively defined as

$$M = \frac{U}{2\mathcal{V}} \sum_{\mathbf{k}} (\langle n_{\mathbf{k}+}\rangle - \langle n_{\mathbf{k}-}\rangle),$$

$$\Delta_{\pm 1}(\mathbf{k}) = -\frac{1}{\mathcal{V}} \sum_{\mathbf{k}'} V_{\mathrm{SC}}(\mathbf{k},\mathbf{k}') \mathcal{F}_\pm(\mathbf{k}',\tau=0). \quad (3)$$

All of the order parameters can be calculated using the above Green's functions. They are found to satisfy

$$M = \frac{U}{2\mathcal{V}} \sum_{\mathbf{k}} \left\{ \frac{\epsilon^\uparrow_{\mathbf{k}}[1 - 2f(E_-)]}{2E_-(\mathbf{k})} - \frac{\epsilon^\downarrow_{\mathbf{k}}[1 - 2f(E_+)]}{2E_+(\mathbf{k})} \right\}, \quad (4)$$

$$\Delta_{\pm 1}(\mathbf{k}) = \frac{-1}{\mathcal{V}} \sum_{\mathbf{k}'} V_{\mathrm{SC}}(\mathbf{k},\mathbf{k}') \frac{1 - 2f[E_\pm(\mathbf{k}')]}{2E_\pm(\mathbf{k}')} \Delta_{\pm 1}(\mathbf{k}'), \quad (5)$$

where $\epsilon^{\uparrow,\downarrow}_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu \pm M$, $E_\pm(\mathbf{k}) = \sqrt{(\epsilon^{\downarrow,\uparrow}_{\mathbf{k}})^2 + |\Delta_{\pm 1}(\mathbf{k})|^2}$, and $f(E)$ is the Fermi function. The chemical potential μ is determined from the equation for the number of electrons per unit volume, or particle density,

$$n = \frac{1}{\mathcal{V}} \sum_{\mathbf{k}} \left\{ 1 - \frac{\epsilon^\uparrow_{\mathbf{k}}[1 - 2f(E_-)]}{2E_-(\mathbf{k})} - \frac{\epsilon^\downarrow_{\mathbf{k}}[1 - 2f(E_+)]}{2E_+(\mathbf{k})} \right\}, \quad (6)$$

which is equal to unity at half-filling.

Equations (4)–(6) with $n=1$ comprise the self-consistent equations for the ferromagnetic superconducting system. We solve the equations for the simple case of a spherical Fermi surface at half-filling. It is convenient to solve these equations by converting the summations over **k** space to continuum integrals over energy,

$$\bar{M} = \frac{\bar{U}}{32\pi^2}\int_0^\infty d\bar{\varepsilon}\int_0^\pi d\theta \sin\theta\sqrt{\bar{\varepsilon}} \times \left\{ \frac{\bar{\varepsilon}^\uparrow \tanh\left[\frac{\bar{E}_-}{2\bar{T}}\right]}{\bar{E}_-} - \frac{\bar{\varepsilon}^\downarrow \tanh\left[\frac{\bar{E}_+}{2\bar{T}}\right]}{\bar{E}_+} \right\}, \tag{7}$$

$$1 = \frac{\bar{V}}{32\pi^2}\int_{\bar{\epsilon}_{F_\pm}-\bar{\omega}_c}^{\bar{\epsilon}_{F_\pm}+\bar{\omega}_c} d\bar{\varepsilon}\int_0^\pi d\theta \left\{ \frac{\sqrt{\bar{\varepsilon}}\cdot\sin^3\theta}{\bar{E}_\pm}\tanh\left[\frac{\bar{E}_\pm}{2\bar{T}}\right] \right\}, \tag{8}$$

$$n = \frac{1}{16\pi^2}\int_0^\infty d\bar{\varepsilon}\int_0^\pi d\theta \sin\theta\sqrt{\bar{\varepsilon}} \times \left\{ 2 - \frac{\bar{\varepsilon}^\uparrow \tanh\left[\frac{\bar{E}_-}{2\bar{T}}\right]}{\bar{E}_-} - \frac{\bar{\varepsilon}^\downarrow \tanh\left[\frac{\bar{E}_+}{2\bar{T}}\right]}{\bar{E}_+} \right\}, \tag{9}$$

where $\bar{\epsilon}_{F_\pm}=\bar{\mu}\pm\bar{M}$, $\bar{\varepsilon}^{\downarrow,\uparrow}=\bar{\varepsilon}-\bar{\epsilon}_{F_\pm}$, and $\bar{E}_\pm = \sqrt{(\bar{\varepsilon}^{\downarrow,\uparrow})^2+\sin^2\theta|\bar{\Delta}_\pm|^2}$. In the above equations, the unit of energy is rescaled by the factor $\frac{\hbar^2 n^{2/3}}{2m^*}$. The dimensionless exchange and pairing interactions $\bar{U}$ and $\bar{V}$ are thus defined by $\bar{U}=U(\frac{\hbar^2 n^{2/3}}{2m^*})^{-1}$ and $\bar{V}=V(\frac{\hbar^2 n^{2/3}}{2m^*})^{-1}$, and the dimensionless energies $\bar{\epsilon}_{F_\pm}$, $\bar{\varepsilon}$, $\bar{\omega}_c$, $\bar{E}_\pm$, $\bar{\Delta}_\pm$, and $\bar{\mu}$ are defined analogously. The dimensionless temperature is defined by $\bar{T} = k_B T(\frac{\hbar^2 n^{2/3}}{2m^*})^{-1}$. We choose $\bar{\omega}_c=0.01\bar{\epsilon}_F$, where $\bar{\epsilon}_F$ is the dimensionless Fermi energy at $\bar{M}=\bar{T}=0$.

By solving the equations self-consistently, we can investigate the interplay between the magnetism and the superconductivity in the coexisting state. This issue was discussed previously based on a similar framework, with the emphasis placed on the effects on the SC pairing due to the critical spin fluctuations in FM compounds.[26] The present work focuses on the reciprocal action, i.e., the influence of the SC on the FM.

According to Stoner theory, a Fermi gas can exhibit ferromagnetism only when the effective FM exchange is larger than the critical exchange strength. For a system described by Eq. (1), U represents the effective exchange interaction strength. In the absence of the p-wave SC interaction, $\bar{V}=0$, the dimensionless critical exchange strength $\bar{U}_c(0)\approx 12.761\,04$. For $\bar{V}\neq 0$, we calculate $\bar{U}_c(\bar{V})$. As shown in Fig. 1, the $\bar{T}=0$ dimensionless critical exchange strength $\bar{U}_c(\bar{V})$ decreases as $\bar{V}$ increases, which implies that the p-wave Cooper pairing reduces the barrier to the onset of the magnetization of the Fermi gas. We note that $\bar{V}$ might be very small in a real system, so the enhancement effect of the superconductivity on the ferromagnetism may be very weak. The inset of Fig. 1 shows the details of $\bar{U}_c(\bar{V})$ in the region of small $\bar{V}$, where the decreasing tendency of $\bar{U}_c(\bar{V})$ with increasing $\bar{V}$ still can be seen clearly.

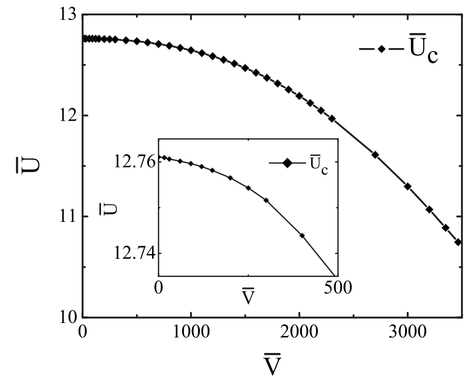

FIG. 1. The dimensionless critical exchange strength $\bar{U}_c(\bar{V})$ as a function of the p-wave interaction strength $\bar{V}$ at $\bar{T}=0$. Inset: enlargement of the region $0\leq\bar{V}\leq 500$.

To further demonstrate the influence of the SC on the FM, we discuss the magnetization $m\equiv\langle n_+\rangle-\langle n_-\rangle$ as a function of $\bar{V}$ at $\bar{T}=0$. Here we use $m=2\bar{M}/\bar{U}$ instead of $\bar{M}$ to eliminate the dependence of $\bar{U}_c$ upon $\bar{V}$. As shown in Fig. 2, $m(\bar{V})$ increases with $\bar{V}$ for each given value of $\bar{U}$. For $\bar{U}>\bar{U}_c(0)$, $m(0)$ is finite, since the system is spontaneously magnetized, and $m(\bar{V})$ increases monotonically from $m(0)$, eventually reaching unity at a finite $\bar{V}\leq 2300$. For $\bar{U}<\bar{U}_c(0)$, however, $m(\bar{V})=0$ for $\bar{V}<\bar{V}_c(\bar{U})$, and then $m(\bar{V})\neq 0$ increases sharply with $\bar{V}$ for $\bar{V}\geq\bar{V}_c(\bar{U})$, eventually reaching unity at $\bar{V}>2300$. The dimensionless critical pairing strength $\bar{V}_c(\bar{U})$ corresponds to the reduction in the dimensionless critical ex-

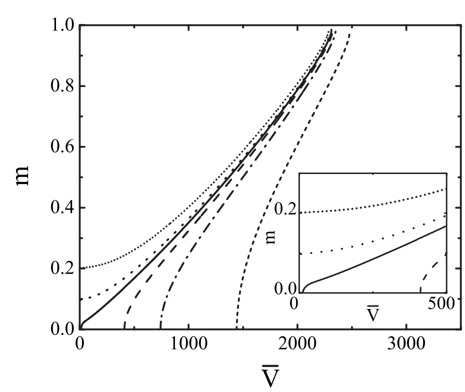

FIG. 2. Plots of the electronic magnetization $m\equiv\langle n_+\rangle-\langle n_-\rangle$ as a function of the dimensionless p-wave interaction strength $\bar{V}$ at $\bar{T}=0$ for fixed values of $\bar{U}$. From larger to smaller m at fixed $\bar{V}$, $\bar{U}=12.8$ (short dotted), 12.77 (dotted), 12.761 (solid), 12.743 (dashed), 12.7 (dash dotted), and 12.495 (short dashed). Inset: enlargement of the region $0\leq\bar{V}\leq 500$.

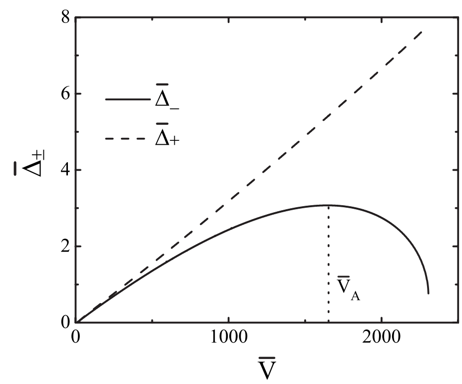

FIG. 3. Plots of $\bar{\Delta}_+$ (dashed) and $\bar{\Delta}_-$ (solid) as functions of $\bar{V}$ at $\bar{U}=12.77$ and $\bar{T}=0$. $\bar{V}_A$ is the value of $\bar{V}$ at which $\bar{\Delta}_-$ has a maximum, and $\bar{\Delta}_- \to 0$ at $\bar{V} \to \sim 2300$, the point at which $m \to 1$ in Fig. 2.

change strength $\bar{U}_c(\bar{V})$ at which the onset of the ferromagnetism is induced, as pictured in Fig. 1. This is a second way in which the p-wave superconductivity can enhance the ferromagnetism.

A similar effect was found in the ferromagnetic spin-1 Bose gas which exhibits two phase transitions, the FM transition and Bose-Einstein condensation (BEC). The BEC temperature increases with FM couplings and, on the other hand, the FM transition is significantly enhanced due to the onset of the BEC.[31] Considering that triplet Cooper pairs behave somewhat like spin-1 bosons, a FM superconductor is analogous to a FM Bose gas.

Figure 3 displays plots of the p-wave SC order parameters, $\bar{\Delta}_\pm$ as functions of $\bar{V}$ at $\bar{T}=0$ and $\bar{U}=12.77$, just above the $\bar{V}=0$ dimensionless critical exchange value $\bar{U}_c(0)$. Although with increasing $\bar{V}$, $\bar{\Delta}_+$ rises monotonically, $\bar{\Delta}_-$ initially rises, reaches a maximum at $\bar{V}_A$, and then decreases at an increasing rate until it vanishes discontinuously when $m(\bar{V})=1$. For $\bar{U}=12.77$, $m(\bar{V})>0$ is shown by the dotted curve in Fig. 2, so that $\bar{\Delta}_+>\bar{\Delta}_-$ for all $\bar{V}$. Since m also grows with $\bar{V}$, the mean number of spin-down electrons decreases with increasing $\bar{V}$, vanishing when $m\to 1$ at $\bar{V}\approx 2300$, at and beyond which $\bar{\Delta}_-\to 0$.

We now discuss the finite-temperature properties of the system. We define $\bar{M}'$ to be the magnetic order parameter when $\bar{V}=0$, for which $\bar{\Delta}_\pm=0$. The $\bar{T}$ dependencies of the order parameters $\bar{\Delta}_\pm$, $\bar{M}$, and $\bar{M}'$ are obtained numerically and shown for $\bar{V}=300$ and three different $\bar{U}$ cases in Fig. 4. The order parameters become nonvanishing below their respective dimensionless transition temperatures $\bar{T}_{c\pm}$, $\bar{T}_m$, and $\bar{T}'_m$. In each case, the SC order parameters $\bar{\Delta}_\pm$ increase monotonically with decreasing $\bar{T}$ below $\bar{T}_{c\pm}$, respectively. In the FM superconductor, $\bar{T}_{c-}<\bar{T}_{c+}$ and $\bar{\Delta}_-(\bar{T})<\bar{\Delta}_+(\bar{T})$, as shown in Figs. 4(a)–4(c). In addition, $\bar{M}'(\bar{T})$ also increases monotonically with decreasing $\bar{T}$ for the ferromagnet in the absence of any superconductivity, as depicted in Figs. 4(a) and 4(b) for the respective cases $\bar{U}>\bar{U}_c(0)$ and $\bar{T}'_m>\bar{T}_{c+}$ and $0<\bar{T}'_m<\bar{T}_{c+}$. However, the $\bar{T}$ dependence of $\bar{M}$ is nontrivial when p-wave superconductivity is present. In the first case pictured in Fig. 4(a), $\bar{M}(\bar{T})=\bar{M}'(\bar{T})$ for $\bar{T}'_m>\bar{T}_{c+}$, as in the absence of superconductivity. However, $\bar{M}(\bar{T})$ exhibits an upward kink at $\bar{T}_{c+}$ below which $\bar{\Delta}_+\neq 0$. Then, for $\bar{T}_{c-}<\bar{T}<\bar{T}_{c+}$, $\bar{M}$ increases sharply with decreasing $\bar{T}$, and exhibits a downward kink at $\bar{T}_{c-}$ below which $\bar{\Delta}_-\neq 0$. Below $\bar{T}_{c-}$, $\bar{M}(\bar{T})$ then decreases monotonically with $\bar{T}$. This case was discussed previously in a similar scenario.[32]

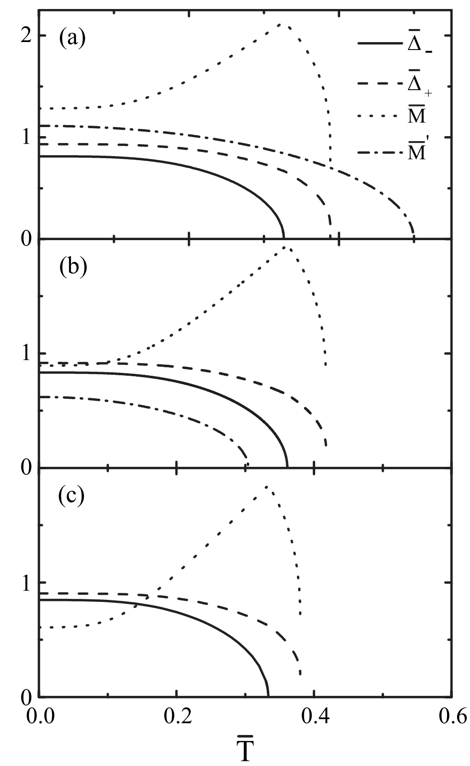

FIG. 4. Shown are plots of the order parameters $\bar{M}$ (dotted), $\bar{\Delta}_+$ (dashed), and $\bar{\Delta}_-$ (solid) as functions of $\bar{T}$ in the coexistence state for $\bar{V}=300$. $\bar{M}'$ (dash-dotted) is the magnetic order parameter when $\bar{V}=0$. (a) $\bar{U}=12.79>\bar{U}_c(0)$ and $\bar{T}'_m>\bar{T}_{c+}$. (b) $\bar{U}=12.77>\bar{U}_c(0)$ but $0<\bar{T}'_m<\bar{T}_{c+}$. (c) $\bar{U}=12.76<\bar{U}_c(0)$ but $\bar{U}>\bar{U}_c(V)$. The ferromagnetism is induced due to the p-wave pairing ($\bar{M}\neq 0$) even though $\bar{M}'=0$.

The case $\bar{T}'_m<\bar{T}_{c\pm}$ not previously discussed is more interesting. Two examples of this case with $\bar{V}=300$ are shown in Figs. 4(b) and 4(c). In Fig. 4(b), the magnetization $\bar{M}'$ for $\bar{V}=0$ (and $\bar{\Delta}_\pm=0$) is so weak that $0<\bar{T}'_m<\bar{T}_{c-}$, but a nonvanishing $\bar{V}$ enhances the magnetization, $\bar{M}$, causing the actual dimensionless Curie temperature $\bar{T}_m$ to equal $\bar{T}_{c+}$, below which both $\bar{\Delta}_+(\bar{T})$ and $\bar{M}(\bar{T})$ become discontinuously nonvanishing, signaling a first-order transition. Their behaviors for $\bar{T}<\bar{T}_{c+}=\bar{T}_m$ are then qualitatively similar to those shown in Fig. 4(a), with $\bar{\Delta}_-(\bar{T})\neq 0$ for $\bar{T}<\bar{T}_{c-}$, causing a downward kink in $\bar{M}(\bar{T})$ at $\bar{T}_{c-}$, below which $\bar{M}(\bar{T})$ decreases monotoni-

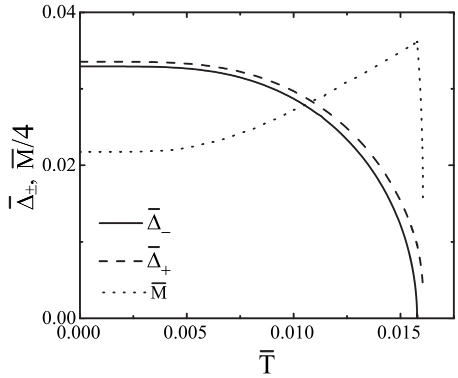

FIG. 5. Plots of the order parameters $\bar{M}$ (dotted), $\bar{\Delta}_+$ (dashed), and $\bar{\Delta}_-$ (solid) as functions of $\bar{T}$ in the coexistence state for $\bar{V}=20$ and $\bar{U}=12.761<\bar{U}_c(0)$.

cally with $\bar{T}$. For the more extreme case when $\bar{U}<\bar{U}_c(0)$ and $\bar{T}_m'=0$ but $\bar{U}>\bar{U}_c(\bar{V})$ depicted in Fig. 4(c), the behaviors of the three order parameters are very similar to those shown in Fig. 4(b).

Considering that $\bar{V}$ is usually small in real systems, a case with $\bar{V}=20$ is checked, as shown in Fig. 5 where $\bar{U}$ is taken to be 12.761, slightly lower than $\bar{U}_c(0)$ but larger than $\bar{U}_c(20)\approx 12.7608$. Figure 5 appears very similar to Fig. 4(c).

Although we did not investigate the limit $\bar{V}\to 0+$, the examples with $\bar{V}=300$ and $\bar{V}=20$ of the case $\bar{T}_m'<\bar{T}_{c+}$ pictured in Figs. 4(b), 4(c), and 5 suggest that in FM superconductors, the actual Curie temperature $\bar{T}_m$ is unlikely to ever be lower than the upper SC transition temperature $\bar{T}_{c+}$, even if the FM order were extremely weak. In other words, these examples argue against the possibility of a FM $\bar{T}$ regime inside the p-wave triplet SC regime, with an actual $\bar{T}_m<\bar{T}_{c+}$. Analogously, it was shown that the ferromagnetic transition never occurs below the Bose-Einstein condensation in the FM spin-1 Bose gas.[31] Moreover, the present results are to some extent consistent with the observed phase diagrams of UGe_2,[7] UIr,[8,9] the low-pressure regime of UCoGe,[18] and with the theoretical discussion of Walker and Samokhin,[33] who argued that the superconductivity only occurs within the FM region. In addition, this scenario is consistent with de Haas van Alphen experiments under pressure on UGe_2.[34]

Very recent experiments on UCoGe under pressure were interpreted as potentially having such a FM regime inside the SC regime near to the FM quantum critical point.[18] However, the dc resistance and ac susceptibility measurements of T_m and T_{c+} could not determine if there were a FM region inside the SC one for pressures just below their extrapolated quantum critical pressure p_c, allowing for a first-order phase transition at the point when $T_m=T_{c+}$, beyond which only a parallel-spin triplet state exists.[18] Mineev argued that in the high-pressure regime $p>p_c$ of UCoGe, there would be two possibilities: one in which the onset of superconductivity would exceed that of the ferromagnetism, and one in which they would be the same.[35] Our results pictured in Figs. 4(b), 4(c), and 5 are consistent with the latter possibility, yielding a first-order transition at T_{c+}, below which p-wave superconductivity and ferromagnetism coexist. Further experiments are encouraged to determine if the FM and SC phase regimes with $0<T_m<T_{c+}$ at fixed pressure actually exist in UCoGe, or whether $T_m\geq T_{c+}$.

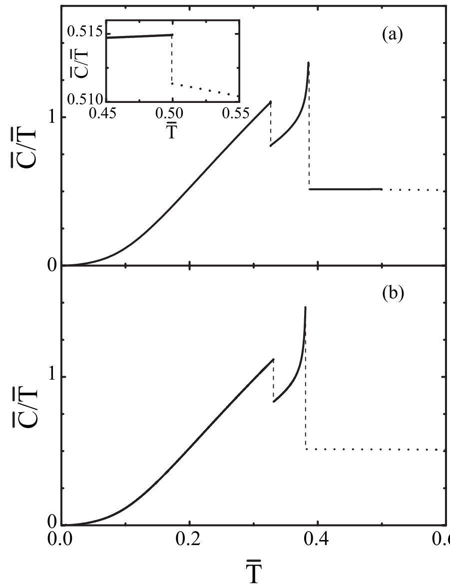

FIG. 6. Plots of the dimensionless electronic specific heat relative to the dimensionless temperature $\bar{C}/\bar{T}$ at constant volume as a function of $\bar{T}$ for (a) a case corresponding to Fig. 4(a). The inset shows a transition from ferromagnetic to paramagnetic phase occurs at the dimensionless ferromagnetic transition temperature $\bar{T}_m\approx 0.5$. The dotted curve denotes the specific heat of the free electron gas; (b) a case corresponding to Fig. 4(b). The discontinuity on the right corresponds to $\bar{T}_m=\bar{T}_{c+}$, at which the transition is first order.

As suggested by the results for the temperature dependencies of the order parameters, the FM superconducting system shows multiple phase transitions, which can be determined experimentally from measurements of the specific heat. Multiple superconducting transitions were first seen in the specific heat of UPt_3,[36] which combined with ultrasound velocity, ultrasonic attenuation, and Knight shift measurements led to the identification of that weakly antiferromagnetic material as a triplet superconductor.[28,29] Multiple transitions were previously suggested for the ferromagnetic superconductors,[37] but the case of weaker ferromagnetism than superconductivity was not discussed, and detailed plots of predictions for experimental tests were not presented. The specific heat at constant volume for our model can be calculated from

$$\bar{C}(\bar{T})=\bar{T}\frac{\partial\bar{S}}{\partial\bar{T}},$$

where the dimensionless electronic contribution to the entropy $\bar{S}=S/k_B$ is given by

$$\bar{S}=-\sum_{\mathbf{k},\sigma=\pm}\{f(\bar{E}_\sigma)\ln f(\bar{E}_\sigma)+[1-f(\bar{E}_\sigma)]\ln[1-f(\bar{E}_\sigma)]\}.$$

The specific heat was calculated previously based on a model of s-wave superconductivity coexisting with ferromagnetism.[38] For *s*-wave superconductors, there is only one SC transition temperature $\bar{T}_c$, at which there is a jump in the specific heat at the second-order transition. However, the case of a *p*-wave superconductor coexisting with ferromagnetism is more interesting. In Figs. 6(a) and 6(b), the results for the specific heat corresponding to the cases pictured in Figs. 4(a) and 4(b) for the order parameters are shown. For the case $\bar{U} > \bar{U}_c$ pictured in Figs. 4(a) and 6(a), there are three phase transitions at the temperatures $\bar{T}_{c-} < \bar{T}_{c+} < \bar{T}_m$. In Fig. 6(b), an example of the case $\bar{T}'_m < \bar{T}_{c+}$ when $\bar{V}=0$ pictured in Fig. 4(b) is shown. In this case with $\bar{V}=300$, there is a first-order phase transition at $\bar{T}_m = \bar{T}_{c+}$, and a second-order phase transition at $\bar{T}_{c-}$.

In conclusion, it is shown that *p*-wave triplet Cooper pairing can enhance the ferromagnetism in superconducting ferromagnets. This enhancement is most prominent for the magnetic exchange interaction U very near to $U_c(0)$, the critical exchange interaction required for the onset of ferromagnetism in the absence of the *p*-wave pairing interaction V. With finite V, $U_c(V)$ is reduced and the ferromagnetic order parameter increases in magnitude with increasing V. The temperature dependencies of the magnetic and parallel-spin superconducting order parameters and of the specific heat are calculated. The results show that the Curie temperature is unlikely to ever be lower than the upper SC transition temperature, in agreement with pressure measurements on UGe_2 and UIr[8,9,34] and upper critical field measurements in the "regular" (non-re-entrant) regime of URhGe.[12] This feature also may be relevant to recent experiments on UCoGe,[18] and suggests that further experiments in the high-pressure phase to determine whether the predicted weak ferromagnetism coexists with the *p*-wave superconductivity. Our results support the possible coexistence of *p*-wave superconductivity with a ferromagnetic state. The temperature dependence of the specific heat exhibits two peaks for weak ferromagnetism in the coexistence state, with a first-order transition at the combined ferromagnetic and upper *p*-wave SC transition, and a lower second-order *p*-wave SC transition. For strong ferromagnetism, the specific heat exhibits three second-order transitions.

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